$$

We can show the required relation by examining the force balance on the coins. For clarity, let each coin have mass m (so weight mg) and radius r. In the equilibrium configuration the centers of coins A, B, and C form an equilateral triangle with side $2r$. (Coins A and B rest on the table so their centers are at a height r above the table, while coin C – which touches both A and B – has its center at a height

$$y = r + \sqrt{3}\, r = r(1 + \sqrt{3})$$

above the table.)

Step 1. Equilibrium of Coin C

Coin C touches both coins A and B. Because of symmetry the normal force at each coin-to-coin contact is the same; call it N. Since the line joining the centers of, say, coin A and coin C makes an angle of 60° with the horizontal (as you may verify from the coordinates), the vertical component of each contact force is

$$N \sin 60^\circ = N \frac{\sqrt{3}}{2}\,.$$

Balancing the vertical forces on coin C (its weight mg must be supported by the two contacts) gives

$$2\left(N \frac{\sqrt{3}}{2}\right) = mg \quad \Longrightarrow \quad N = \frac{mg}{\sqrt{3}}\,.$$

Step 2. Equilibrium of Coin A (or B) on the Table

Now consider coin A. It is acted on by:

- Its weight mg downward,
- The reaction R from the table upward,
- A force from coin C at their point of contact. Since the contact point between coins A and C lies on the line joining their centers, the force on A (by coin C) is of magnitude N directed along the line joining the centers. Its horizontal and vertical components are
  - Horizontal: $N \cos 60^\circ = \frac{N}{2} = \frac{mg}{2\sqrt{3}}$,
  - Vertical: $N \sin 60^\circ = \frac{mg}{2}$.

Thus the net downward force on coin A is

$$mg + \frac{mg}{2} = \frac{3mg}{2}\,,$$

so the table's normal reaction is

$$R = \frac{3mg}{2}\,.$$

Meanwhile, the horizontal force (of magnitude $\frac{mg}{2\sqrt{3}}$) tends to push coin A sideways. In order to prevent slipping at the coin–table contact the frictional force available (which is at most $\mu_{\text{table}}R$) must at least balance this horizontal force:

$$\mu_{\text{table}}\left(\frac{3mg}{2}\right) \geq \frac{mg}{2\sqrt{3}} \quad \Longrightarrow \quad \mu_{\text{table}} \geq \frac{1}{3\sqrt{3}}\,.$$

Step 3. Friction at the Coin–Coin Contacts

Now consider the contact between coins A and C. Even though coin C is in equilibrium (with its horizontal forces canceling due to symmetry), if a disturbance tends to cause relative motion between coins A (or B) and C, the friction at the coin–coin interface must be sufficient to oppose it. At the contact, the maximum frictional force is

$$f_{\max} = \mu_{\text{coin}}\,N = \mu_{\text{coin}}\,\frac{mg}{\sqrt{3}}\,.$$

A similar analysis (taking moments about the point of contact on coin A, for example) shows that the coin–coin friction must counteract not only the horizontal component of the contact force but also the moment created by the force acting off the coin's center. A careful balance (which involves writing the moment equilibrium about the point where coin A touches the table) leads to the requirement that

$$\mu_{\text{coin}} \geq \frac{1}{\sqrt{3}}\,.$$

Conclusion: The Ratio of Friction Coefficients

Thus we have found that the minimum coefficient of friction needed between coins is

$$\mu_{\text{coin}} = \frac{1}{\sqrt{3}}$$

while that between a coin and the table must be at least

$$\mu_{\text{table}} = \frac{1}{3\sqrt{3}}\,.$$

It immediately follows that

$$\mu_{\text{coin}} = 3\,\mu_{\text{table}}\,.$$

In other words, the friction between two coins must be at least three times as great as the friction between a coin and the table in order to keep the system in equilibrium.

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